ROTHAMSTED RESEARCH

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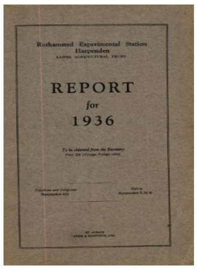

Notes on the Summary Tables

Rothamsted Research

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Notes on the Construction and Use of the Summary Tables.

The presentation of the results of simple experiments is an easy matter, it being usually sufficient to give the mean yields of the individual treatments with an associated standard error by which differences may be compared ; a difference of three times the standard error of a treatment mean may be regarded as significant. In the case of complex or *factorial* experiments, however, where there are all combinations of several sets of treatments, or other factors, the mere presentation of the mean yields of the sets of plots receiving all the different combinations of treatments does not give an adequate or easily comprehended survey of the results.

In order to illustrate the points involved we will first consider the simple type of factorial design in which there are all combinations of two standard fertilisers, nitrogen and phosphate, each at one level in addition to no application. This is called a 2×2 design, and involves the four treatment combinations.

$$(1), n, p, np,$$

the symbol (1) being used to denote no treatment. Each treatment combination will be replicated several times, using a randomised block or Latin square layout. In what follows the symbols are taken to represent the mean yields of each particular combination of treatments.

There are two responses to n, one in the absence of p, namely $(n-(1))$, and one in the presence of p, namely $(np-p)$. These two responses may differ, but frequently the difference is small—too small to be distinguished from experimental error—and in such cases it is often sufficient in considering the results of the experiment to take the average response to n when p is both present and absent. This average response, or *main effect*, is clearly

$$N=\tfrac{1}{2}\left[(np-p)+(n-(1))\right]=\tfrac{1}{2}[np-p+n-(1)]=\tfrac{1}{2}[n-(1)]\ [p+(1)].$$

The advantage of the use of (1) instead of 0 to denote no treatment is that it makes possible the above very simple formal algebraic statement.

The differential response to n in the presence and absence of p is the difference between the response to n when p is present, and the response when p is absent. In the tables of the reports for **1934** and all previous years this difference,

$$(np-p)-(n-(1))=np-p-n+(1),$$

has been called the *interaction* between n and p. In reports for the year **1935** onwards (i.e. beginning with the present report), the interaction has been redefined as *one half* the above difference, i.e. in symbols by

$$N.P=\tfrac{1}{2}[(np-p)-(n-(1))]=\tfrac{1}{2}[np-p-n+(1)]=\tfrac{1}{2}[n-(1)]\ [p-(1)].$$

Note that the differential response to n in the presence and absence of p is the same as the differential response to p in the presence and absence of n, i.e., there is only one interaction between n and p.

The introduction of the factor $\tfrac{1}{2}$ has the following advantages. First the standard errors of the main effects and all interactions of any $2\times2\times2\times$ . . . design are then equal, and secondly the response to any treatment in association with any combination of the other treatments is expressible as the sum or difference of the various main effects and interactions, without any numerical factors. Thus in a 2×2 design the following relations hold :

Response to	Expression in Terms of Treatment Combinations	Expression in Terms of Main Effects and Interactions
n (mean over all p) . .	$\frac{1}{2}[np+n—p—(1)]$	N
n (p absent)	$n—(1)$	$N—N.P$
n (p present)	$np—p$	$N+N.P$
n and p together	$np—(1)$	$N+P$

Similar expressions will hold for any other 2×2 design.

It should be particularly noted that the interaction does not enter into the expression for the response to n and p applied together.

Since the main effects and interactions are statistically independent the standard error of the sum or difference of two of them is $\sqrt{2}$ times the standard error of each.

Example. Peas, Biggleswade, **1933**. The mean yields (ignoring slag, which produced no apparent effect) were (in cwt. per acre) :

	(1)	n	k	nk	
cwt. per acre :	33.0	38.0	32.0	34.1	±1.00

The main effects and interactions are therefore :

$$\left.\begin{array}{r} N\quad 3.6 \\ K—2.4 \\ N.K—1.4 \end{array}\right\} \pm1.00$$

There is a significant response to nitrogen and a significant depression with potash, the interaction not being significant. If the interaction, though not significant is not assumed non-existent, the estimate of the response to n alone is

$$N—N.K=n—(1)=+5.0 \quad \pm1.41.$$

The estimate of the response to the two fertilisers together is

$$N+K=nk—(1)=+1.2 \quad \pm1.41.$$

The $2\times2\times2$ arrangement is similar. The eight treatment combinations are

$$(1),\ n,\ p,\ k,\ np,\ nk,\ pk,\ npk.$$

The main effect of n is the average of the four responses and is therefore

$$N=\tfrac{1}{4}[npk—pk)+(np—p)+(nk—k)+(n—(1))]=\tfrac{1}{4}[n—(1)]\ [p+(1)]\ [k+(1)].$$

The *first order* interaction between N and P is defined as the average of the interactions between N and P in the presence and absence of K, and is therefore

$$N.P=\tfrac{1}{2}[\tfrac{1}{2}(npk—nk—pk+k)+\tfrac{1}{2}(np—n—p+(1))]=\tfrac{1}{4}[n—(1)]\ [p—(1)]\ [k+(1)],$$

and the *second order* interaction is defined as *one half* the difference of the above two interactions, and is therefore

$$N.P.K=\tfrac{1}{2}[\tfrac{1}{2}(npk—nk—pk+k)—\tfrac{1}{2}(np—n—p+(1))]=\tfrac{1}{4}[n—(1)]\ \ [p—(1)]\ \ [k—(1)].$$

Just as there is only one interaction between two treatments, so there are three first order interactions between three treatments, one between each of the pairs of the treatments, but only one second order interaction between the three treatments.

The following expressions for various typical responses may be noted :

Response to :	Expression in Terms of Treatment Combinations	Expression in Terms of Main Effects and Interactions
n (p absent, mean of k and no k)	$\frac{1}{2}[nk+n—k—(1)]$	$N—N.P$
n (p and k absent)	$n—(1)$	$N—N.P—N.K$ $+N.P.K$
n and p (mean of k and no k) . .	$\frac{1}{2}[npk+np—k—(1)]$	$N+P$
n and p (k absent)	$np—(1)$	$N+P—N.K—P.K$
n, p and k (complete fertiliser)	$npk—(1)$	$N+P+K+N.P.K$

If the second order interaction is ignored the response to all three factors in conjunction is equal to the sum of the main effects of the three factors.

When three levels of a fertiliser are included the situation is somewhat more complicated. If the yields at no single and double dressing are n_0, n_1, n_2 the response to the double dressing, which may be defined as the *linear response,* is measured by

$$N_1 = n_2 - n_0,$$

and the excess of the response to the second dressing over the response to the first, which may be defined as the *curvature* of the response curve, is measured by

$$N_2 = (n_2 - n_1) - (n_1 - n_0) = n_2 - 2n_1 + n_0.$$

With the ordinary type of fertiliser response curve the curvature will in general be negative.

With this convention the response to the single dressing is given by

$$n_1 - n_0 = \tfrac{1}{2}(N_1 - N_2),$$

and the additional response to the double dressing is given by

$$n_2 - n_1 = \tfrac{1}{2}(N_1 + N_2).$$

With two fertilisers each at three levels the linear response and curvature to each fertiliser will be the mean of such responses over all three levels of the other fertiliser. The *interaction of the linear responses* will be defined as

$$N_1.P_1 = \tfrac{1}{2}(n_2p_2 - n_2p_0 - n_0p_2 + n_0p_0) = \tfrac{1}{2}(n_2 - n_0)\ (p_2 - p_0).$$

(The factor $\tfrac{1}{2}$ is omitted in the tables given in the 1934 report.) The other three components of interaction may be defined similarly, but in a first study of the results of 3×3 fertiliser experiments it is usually sufficient to confine attention to the above component of interaction. In $3\times3\times3$ experiments the *second order interaction of linear responses,* namely

$$N_1.P_1.K_1 = \tfrac{1}{4}(n_2p_2k_2 - n_2p_2k_0 - n_2p_0k_2 - n_0p_2k_2 + n_0p_0k_2 + n_0p_2k_0 + n_2p_0k_0 - n_0p_0k_0)$$
$$= \tfrac{1}{4}(n_2 - n_0)\ (p_2 - p_0)\ (k_2 - k_0),$$

may be of interest.

The summaries of this report are so arranged that as far as possible the main effects and first order interactions are available without the necessity of taking out any means. The first order interactions are often given in the form of response to one treatment in the presence of, and in the absence of the other, under the heading of " differential responses." The standard errors (prefaced by the sign $\pm$) applicable to all comparisons which are likely to be of interest are also shown. They are deduced from the standard errors per plot, which are given in the details of the experiment.

The rough rule for use with standard errors is that a quantity is significant if it is greater than twice its standard error, and the difference between two quantities having the same standard error is significant if it is three times that standard error. Thus the mean response to sulphate of ammonia in the 1933 Brussels Sprouts experiment at Woburn is given as 9.01 cwt. $\pm$1.89 cwt., which is therefore significant, since the response is almost 5 times its standard error. The responses in the absence and presence of poultry manure are 12.38 cwt. and 5.64 cwt., each with a standard error of $\pm$2.67, and the differential response (or interaction) which is the difference of these, though suggestive, is not significant, being only about two and a half times the standard error of each of them. The response to sulphate of ammonia in the presence of poultry manure, 5.64, is significant, being more than twice its standard error. The same interaction can be looked at from the point of view of response to poultry manure in the absence and presence of sulphate of ammonia. These responses

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are 8.18 and 1.44 cwt., again with a standard error of ± 2.67, giving a mean response of 4.81 cwt. with a standard error of ± 1.89. The mean response and the response in the absence of sulphate of ammonia are therefore significant, but the response in the presence of sulphate of ammonia is small and not significant. We have here a case of common occurrence where one of two quantities is significant and the other is not, but where the two quantities do not differ significantly from one another.

Standard errors, besides their use for testing the significance of comparisons from one particular experiment, are of importance when the results of a number of experiments are combined, since they serve as a measure of the reliability of each experiment, and also give the information necessary for telling whether the variation from experiment to experiment in the effect under survey is a real one or whether it can be attributed to experimental errors.

The second and higher order interactions are likely to be of even less importance than the first order interactions, and this fact is made use of in confounding, which is a modification of the randomised block method, introduced in order to keep the number of plots per block small while allowing a large number of different treatments. In confounded experiments certain comparisons representing high order interactions are confounded (i.e. mixed up) with differences between blocks. Thus in the $2\times2\times2$ arrangements given above, the plots receiving the treatments *npk*, *n*, *p* and *k* might be put in one set of sub-blocks of 4 plots, and the plots receiving treatments *np*, *nk*, *pk* and (1) in another set of sub-blocks of 4 plots. The second order interaction would then be completely confounded. On irregular land a considerable increase of precision may result from keeping the blocks small. There are many examples of confounding of varying complexity in the experiments of this report. There is not space to discuss all the implications of confounding here, but it will be seen that in general the results of interest, namely the main effects and first order interactions, are unaffected by confounding, and tables involving these interactions only can be used without regard to the confounding. In certain cases, e.g., $3\times2\times2$ and $3\times3\times2$ experiments, where some of the first order interactions are unavoidably slightly confounded, these interactions have slightly higher standard errors than the others ; this is indicated in the tables themselves, the correct standard errors being given.

The higher order interactions are not only unimportant, but it can often be confidently predicted that they are likely to be very small in magnitude compared with the experimental errors. They can therefore be used to provide an estimate of experimental error instead of the usual estimate provided by replication. This makes possible complex experiments in which each combination of treatments occurs once only, thus enabling greater complexity to be attained with a reasonable number of plots. The 1933 potato experiment at Wisbech is an example of this type of layout.

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